Spin-wave theory of two-dimensional ferromagnets in the presence of dipolar interactions and magnetocrystalline anisotropy

Patrick Bruno*
Institut für Angewandte Physik, Universität Regensburg, D-8400 Regensburg, Federal Republic of Germany
(Received 9 August 1990)

A spin-wave theory of two-dimensional ferromagnets taking into account both the long-range dipolar interactions and a uniaxial magnetocrystalline anisotropy is presented. We show that in the case of the perpendicular easy axis the stabilization of long-range ferromagnetic order at finite temperature is due mainly to the anisotropy-induced gap at the bottom of the spin-wave spectrum, whereas the dipolar interactions play a negligible role; on the other hand, in the case of easy-plane anisotropy, the spin-wave spectrum remains gapless, so that the stabilization of long-range ferromagnetic order is due in turn to the long-range character of the dipolar interactions. The temperature dependence of the magnetization is calculated in both cases. The results are discussed in connection with experimental data reported for fcc (001) cobalt monolayers epitaxially grown onto (001) copper single crystals.

I. INTRODUCTION

For more than half a century, the problem of two-dimensional ferromagnetism has been a matter of controversy. During the last years, it has attracted a renewed interest: due to the development of ultrahigh-vacuum technology, one is able to prepare epitaxial monolayers of high structural quality, while their magnetic properties can be investigated by means of a number of new techniques.

When Bloch, in 1930, introduced for the first time the concept of the spin wave, he had already pointed out that a two-dimensional spin system described by a Heisenberg Hamiltonian cannot present any spontaneous magnetization at finite temperature;\(^1\) Bloch's argument is that, as the spin-wave dispersion relation is \(E \approx k^2\) in the neighborhood of \(k = 0\), the spin-wave density of states \(N(E)\) in a two-dimensional system is independent of \(E\) for \(E\) close to zero. Thus, the magnetization variation due to the thermal excitator of spin waves is

\[
\Delta M(T) \sim \int_0^1 \frac{N(E) dE}{\exp(E/k_B T) - 1} \sim k_B \int_0^{+\infty} \frac{dx}{\exp(x) - 1}.
\]

The integral in Eq. (1) diverges logarithmically at its lower bound, suggesting that the magnetization must be zero at finite temperature. Bloch's conjecture was later rigorously proved by Mermin and Wagner,\(^2\) whose theorem shows that any long-range magnetic order at finite temperature cannot exist in a two-dimensional system, provided that (i) the interaction between the spins is isotropic and that (ii) the interaction is short ranged (more precisely, it must decrease more rapidly than \(1/r^3\)). On the other hand, it is now experimentally established that long-range ferromagnetic order can exist at finite temperature in clean monolayers of high structural quality.\(^3,4\) The apparent contradiction between theoretical predictions and experimental results arises from the fact that, as one can easily see, none of the hypotheses of the Mermin-Wagner theorem is fulfilled by a real physical system: (i) owing to its low symmetry, a two-dimensional system is intrinsically anisotropic, and (ii) in a spin system long-ranged interactions always exist (the magnetic dipole-dipole interactions, which decrease like \(1/r^3\)). In order to discuss the existence or nonexistence of long-range ferromagnetic order in two-dimensional systems, it is therefore necessary to take into account these departures from Mermin and Wagner's ideal situation.

It has been pointed out by Herring and Kittel\(^5\) that a uniaxial anisotropy favoring an easy magnetization axis removes the logarithmic divergence by opening a gap at the bottom of the spin-wave spectrum and therefore stabilizes the long-range ferromagnetic order at finite temperature; this has also been discussed by Doring\(^6\) and Corciovei,\(^7\) among others. It is worth emphasizing that the above argument does not hold in the case of a uniaxial anisotropy favoring an easy-magnetization plane; in this case the spins can be freely rotated in the easy plane and no gap appears in the spin-wave spectrum. As a consequence the anisotropy arising from dipolar interactions (shape anisotropy) is not able to stabilize the long-range magnetic order.

The influence of the long-range character of the dipolar interactions has been studied by Maleev\(^8\) and by Yafet et al.;\(^9\) they found that the long-range character of the dipolar interactions removes the logarithmic divergence and that long-range ferromagnetic order actually exists at finite temperature.

Up to now, there has been no theoretical investigation taking into account both the anisotropy and the dipolar interactions. Their respective influence on the magnetization therefore remains to be clarified. The goal of the present paper is to study the respective role of anisotropy and dipolar interactions in the case of a uniaxial anisotro-
The magnetic behavior of the system results from a subtle interplay between dipolar interactions and anisotropy, which leads to striking results.

(i) In the case where the effective anisotropy (resulting from the balance between magnetocrystalline and dipolar anisotropies) leads to an easy axis perpendicular to the film plane, there occurs a gap at the bottom of the spin-wave spectrum, stabilizing the magnetization against thermally excited spin waves; in that case the long-range character of dipolar interactions turns out to play a negligible role.

(ii) The situation is completely different if the effective anisotropy leads to an easy plane. In that case, there is no gap in the spin-wave spectrum and the magnetization stabilization results from a square-root modification of the spin-wave dispersion relation at large wavelengths which is, in turn, a direct consequence of the long-range character of dipolar interactions.

In both cases, we have evaluated the temperature dependence of the magnetization and compared it with experimental data.

Moreover, we point out that for a given range of anisotropy energy, the uniformly magnetized state becomes unstable because of the occurrence of soft spin-wave modes and that the system spontaneously develops some ripplelike fluctuations of the spin direction.

The paper is organized as follows. The Hamiltonian is described in Sec. II, and the expression of the magnetization is derived in Sec. III using the Holstein-Primakov theory. Section IV is devoted to the discussion of the stability of the homogeneously magnetized state. Then we calculate the magnetization in the ground state and at finite temperature, respectively, in Sec. V and VI. The results are discussed and compared with experimental data in Sec. VII with emphasis on the Co(O01)/Cu(O01) system.

II. THE HAMILTONIAN

We consider a two-dimensional square array of spins $S$ and lattice parameter $a$. The exchange interaction is described by a Heisenberg Hamiltonian:

$$\mathcal{H}_e = -2J \sum_{\langle i,j \rangle} S_i \cdot S_j .$$ (2)

We introduce the magnetocrystalline anisotropy as a short-ranged pseudodipolar interaction

$$\mathcal{H}_K = -K \sum_{\langle i,j \rangle} \left[ \frac{1}{3} \langle S_i \cdot S_j - 3(S_i \cdot u_{ij})(S_j \cdot u_{ij}) \rangle \right] .$$ (3)

Such an anisotropy automatically matches the symmetry of the underlying lattice, and it is easily checked that it results in a uniaxial anisotropy. The dipolar interaction takes the usual form

$$\mathcal{H}_d = \frac{1}{3} \sum_{\langle i,j \rangle} \frac{(2\mu_B)^2}{r_{ij}^3} \left[ S_i \cdot S_j - 3(S_i \cdot u_{ij})(S_j \cdot u_{ij}) \right] .$$ (4)

In Eqs. (2) and (3), the symbol $\langle i,j \rangle$ indicates that the sums are restricted to nearest-neighbor pairs, and the sign convention contained in Eq. (3) implies that positive (negative) values of $K$ favor the magnetization lying perpendicular to the film plane (in the plane). The strength of the different terms is best expressed in equivalent magnetic field units; we therefore introduce the exchange, anisotropy, and dipolar fields given, respectively, by

$$2\mu_B H_{ex} = 2JS ,$$ (5)

$$2\mu_B H_K = 6KS ,$$ (6)

$$H_d = 4\pi M_0 \beta ,$$ (7)

where $M_0 = 2\mu_B S\sqrt{2}/a^3$ is the saturation magnetization, and the numerical factor $\beta = 0.762$ reflects the discreteness of the lattice. Note that, in order to unambiguously define the magnetization in a two-dimensional system, one has to assign a finite thickness to the system; the most natural way is to assign to the monolayer the thickness it would have in the bulk material from which it is cut. With this definition for a given magnetic moment per atom the monolayer has the same magnetization as the bulk material. Our definition of $M_0$ is for a fcc (001) plane; the value of $\beta$ is, of course, also dependent on this choice (but not the product $M_0 \beta$, which is the quantity of physical interest). The physical behavior of the system is determined by the respective orders of magnitude of $H_{ex}$, $H_d$, and $H_K$. For a cobalt fcc (001) monolayer, the dipolar field amounts to $H_d \approx 14$ kOe (assuming the bulk magnetization $4\pi M_0 = 18$ kOe). The exchange energy $J$ in monolayers is not known, either experimentally or theoretically; however, it seems reasonable to assume the bulk value as a first approximation. We estimate the latter by using the Rushbrooke-Wood mnemonic formula which gives a fairly good approximation for the transition temperature of a three-dimensional Heisenberg ferromagnet

$$T_c = k_B T_c/J = \frac{5}{6}(z-1)[11S(S+1)-1] ,$$ (8)

where $T_c$ is the Curie temperature and $z$ the number of nearest neighbors (for bulk Co $T_c = 1388$ K and $z = 12$, respectively). This estimate yields $H_{ex} = 1870$ kOe for the exchange field. The value of the anisotropy field $H_K$ can range between $-100$ and $+100$ kOe in monolayers, depending on the system in consideration. Since the dipolar interactions contribute to some anisotropy via the demagnetizing effect, the easy-magnetization axis is determined by the sign of the effective anisotropy field defined by

$$H_{d}^* = H_K - H_d .$$ (9)

For positive (negative) values of the effective anisotropy field, the magnetization lies perpendicularly to the plane (in the plane); we shall hereafter refer to these two cases as the perpendicular-easy-axis and easy-plane anisotropy,
III. SPIN WAVES

We then introduce the Holstein-Primakov boson creation and annihilation operators \( a_s^* \) and \( a_s \), respectively,\(^{13}\) make the usual linear approximation (noninteracting spin waves), and perform a two-dimensional Fourier transformation. Within an unimportant additive constant, the Hamiltonian is now written

\[
\mathcal{H} = \sum_q \left[ A_q a_q^* a_q + \frac{1}{2} (B_q a_q a_{-q} + B_q^* a_q^* a_{-q}^*) \right] 
\]

with

\[
A_q = A_q^x + A_q^y + A_q^z 
\]

and

\[
B_q = B_q^x + B_q^y + B_q^z 
\]

where \( q = k a \) is the reduced two-dimensional wave vector. The exchange matrix elements \( A_q^x \) and \( B_q^x \) are given by

\[
A_q^x = 2 \mu_B H_{ex} (4 - 2 \cos q_x - 2 \cos q_y), \\
B_q^x = 0 
\]

The matrix elements of the magnetocrystalline anisotropy are

\[
A_q^y = 2 \mu_B H_{Kx} (4 \cos q_x + 4 \cos q_y) / 6, \\
B_q^y = 2 \mu_B H_{Ky} (\cos q_x - \cos q_y), 
\]

in the case of perpendicular easy axis, and

\[
A_q^z = -2 \mu_B (H_{Kz} / 2)(2 + 2 \cos q_x - \cos q_y), \\
B_q^z = 2 \mu_B (H_{Kz} / 2) \cos q_y, 
\]

in the case of easy-plane anisotropy. For the dipolar interaction, the calculation of the matrix elements is more complicated because of its long-range character; we have used a method similar to the one described in Ref. 9. We thus obtain (within terms of the order of \( q^2 \))

\[
A_q^x = -2 \mu_B \left[ H_d / 2 \right] \left[ 1 - q / (4a) \right], \\
B_q^x = 2 \mu_B \left[ H_d / (4a) \right] \exp(-2i\phi_q), \\
A_q^y = 2 \mu_B \left[ H_d / 2 \right] \left[ 1 + \cos(2\phi_q) \right] q / (4a), \\
B_q^y = 2 \mu_B \left[ H_d / 2 \right] \left[ 1 - \cos(2\phi_q) \right] q / (4a), \\
A_q^z = 2 \mu_B \left[ H_d / 2 \right] \left[ 1 - \cos(2\phi_q) \right] q / (4a), \\
B_q^z = 2 \mu_B \left[ H_d / 2 \right] \left[ 3 - \cos(2\phi_q) \right] q / (4a), \\
\]

respectively. For convenience, we shall use two different coordinate systems: the system \((x,y,z)\) is related to the lattice, the \( z \) axis being perpendicular to the film plane; the system \((X,Y,Z)\) refers to the spins, with \( Z \) being the quantization axis. For the perpendicular easy axis the two systems coincide, whereas for easy-plane anisotropy, \( Z \) coincides with \( X \) (with \( Y \) and \( Y \) with \( Z \)).

The energy of the spin waves is given by

\[
\hbar \omega_q = (A_q^z - |B_q|^2)^{1/2}, 
\]

whereas the associated spin deviation is

\[
D_q = A_q^z / (\hbar \omega_q). 
\]

The fact that the spin deviation \( D_q \) is larger than 1 is related to the ellipticity of the spin precession. The expression of the magnetic moment per atom is

\[
m(T) = m_0 - \Delta m_0 - \Delta m(T) 
\]

The first term in Eq. (20) is simply the saturation moment, the second one expresses the ground-state spin reduction due to the zero-point fluctuations (a phenomenon analogous to the one occurring in antiferromagnets) and the third term is the spin deviation due to thermal excitation of spin waves.

It is quite a simple matter to perform numerical calculations of the magnetization, using the above formulas; however, before doing so we aim to obtain some approximated analytical expressions which, although less accurate, are more useful for the physical interpretation of the phenomena. In Secs. IV–VI, we will make use of the following approximated expressions for \( A_q \) and \( B_q \), valid for small \( q \):

\[
A_q = 2 \mu_B \left[ H_{Kx} + H_d q / (4a) \right], \\
B_q = 2 \mu_B H_d q / (4a), \\
A_q^d = 2 \mu_B \left[ H_d / 2 \right] \left[ 1 - \cos(2\phi_q) \right] q / (4a), \\
B_q^d = 2 \mu_B \left[ H_d / 2 \right] \left[ 3 - \cos(2\phi_q) \right] q / (4a), \\
\]

in the case of easy-plane anisotropy. A characteristic feature of the Hamiltonian (10) is the occurrence of terms \( a_q a_{-q} \) and \( a_{-q}^* a_{-q}^* \). As a consequence, the spin waves are not circularly but elliptically polarized. The diagonalization was carried out first by Holstein and Primakov.\(^{13}\) The energy of the spin waves is given by

\[
\hbar \omega_q = (A_q^z - |B_q|^2)^{1/2}, 
\]

The fact that the spin deviation \( D_q \) is larger than 1 is related to the ellipticity of the spin precession. The expression of the magnetic moment per atom is

\[
m(T) = m_0 - \Delta m_0 - \Delta m(T) 
\]

The first term in Eq. (20) is simply the saturation moment, the second one expresses the ground-state spin reduction due to the zero-point fluctuations (a phenomenon analogous to the one occurring in antiferromagnets) and the third term is the spin deviation due to thermal excitation of spin waves.

It is quite a simple matter to perform numerical calculations of the magnetization, using the above formulas; however, before doing so we aim to obtain some approximated analytical expressions which, although less accurate, are more useful for the physical interpretation of the phenomena. In Secs. IV–VI, we will make use of the following approximated expressions for \( A_q \) and \( B_q \), valid for small \( q \):

\[
A_q = 2 \mu_B \left[ H_{Kx} + H_d q / (4a) \right], \\
B_q = 2 \mu_B H_d q / (4a), \\
A_q^d = 2 \mu_B \left[ H_d / 2 \right] \left[ 1 - \cos(2\phi_q) \right] q / (4a), \\
B_q^d = 2 \mu_B \left[ H_d / 2 \right] \left[ 3 - \cos(2\phi_q) \right] q / (4a), \\
\]

in the case of easy-plane anisotropy. A characteristic feature of the Hamiltonian (10) is the occurrence of terms \( a_q a_{-q} \) and \( a_{-q}^* a_{-q}^* \). As a consequence, the spin waves are not circularly but elliptically polarized. The diagonalization was carried out first by Holstein and Primakov.\(^{13}\) The energy of the spin waves is given by

\[
\hbar \omega_q = (A_q^z - |B_q|^2)^{1/2}, 
\]

whereas the associated spin deviation is

\[
D_q = A_q^z / (\hbar \omega_q). 
\]

The fact that the spin deviation \( D_q \) is larger than 1 is related to the ellipticity of the spin precession. The expression of the magnetic moment per atom is

\[
m(T) = m_0 - \Delta m_0 - \Delta m(T) 
\]

The first term in Eq. (20) is simply the saturation moment, the second one expresses the ground-state spin reduction due to the zero-point fluctuations (a phenomenon analogous to the one occurring in antiferromagnets) and the third term is the spin deviation due to thermal excitation of spin waves.

It is quite a simple matter to perform numerical calculations of the magnetization, using the above formulas; however, before doing so we aim to obtain some approximated analytical expressions which, although less accurate, are more useful for the physical interpretation of the phenomena. In Secs. IV–VI, we will make use of the following approximated expressions for \( A_q \) and \( B_q \), valid for small \( q \):

\[
A_q = 2 \mu_B \left[ H_{Kx} + H_d q / (4a) \right], \\
B_q = 2 \mu_B H_d q / (4a), \\
A_q^d = 2 \mu_B \left[ H_d / 2 \right] \left[ 1 - \cos(2\phi_q) \right] q / (4a), \\
B_q^d = 2 \mu_B \left[ H_d / 2 \right] \left[ 3 - \cos(2\phi_q) \right] q / (4a), \\
\]
IV. DISCUSSION OF THE STABILITY OF THE HOMOGENEOUSLY MAGNETIZED STATE

As pointed out by Keffer, the stability of the homogeneously magnetized state requires the fulfillment of criterion for every wave vector \( \mathbf{g} \).

In the case of perpendicular anisotropy, it is evident from Eqs. (24a) and (24b) that the condition (26) is satisfied, provided that \( H_{\text{eff}} > 0 \), which is precisely the condition for having a perpendicular easy axis.

For the case of easy-plane anisotropy (\( H_{\text{eff}} < 0 \)), on the other hand, one easily finds from Eqs. (25a) and (25b) that the stability criterion is

\[
|H_{\text{eff}}^K| > H_{\text{crit}},
\]

where the critical effective anisotropy \( H_{\text{crit}} \) is given by

\[
H_{\text{crit}} = H_d^2/(16a^2H_{\text{ex}}).
\]

For \( |H_{\text{eff}}^K| = H_{\text{crit}} \) there is a soft mode (\( \hbar \omega_q = 0 \)) at \( q = q_{\text{crit}} \) with

\[
q_{\text{crit}} = H_d/(4aH_{\text{ex}}),
\]

and the associated spin deviation \( D_z \) diverges at \( q_{\text{crit}} \). For absolute values of the effective anisotropy field smaller than \( H_{\text{crit}} \) there is a range of unstable modes around \( q_{\text{crit}} \), and the homogeneously magnetized state becomes unstable. For cobalt the order of magnitude of the critical effective anisotropy field and critical wave vector are, respectively, \( H_{\text{crit}} \approx 6 \text{ Oe} \) and \( a/q_{\text{crit}} \approx 1400 \text{ A} \).

This instability occurs because, as seen from Eq. (17a), the restoring torque due to dipolar interactions is smaller at finite wave vector than for the homogeneous mode (\( q = 0 \)). Although the magnetocrystalline anisotropy is not large enough to homogeneously pull the magnetization out of the film plane, it is able to induce a ripplelike fluctuation of the magnetization direction of wavelength \( 2\pi a/q_{\text{crit}} \).

This ripplelike instability happens therefore only if the magnetocrystalline anisotropy almost equals the dipolar anisotropy (within a few Oe); since both are of the order of several tens of kOe, this coincidence is very unlikely to happen in experimental situations. In the following, we will restrict ourselves to cases where the effective anisotropy field \( H_{\text{eff}}^K \) (either positive or negative) is in absolute value much larger than \( H_{\text{crit}} \).

V. GROUND-STATE MAGNETIZATION

As pointed out in Sec. II, the ground-state magnetization is, in general, not equal to the saturation magnetization. The spin reduction due to zero-point fluctuations is given by formula (22). We have performed the \( q \)-space integration by replacing the first Brillouin zone by a disk of equal area, and retained only the most significant terms. We thus obtain

\[
\Delta m_0/m_0 = 1/(256\pi^3S^2/\alpha^3)(H_d/H_{\text{ex}})^3 \ln(H_{\text{ex}}/H_{\text{eff}}^K),
\]

in the case of perpendicular easy axis, and

\[
\Delta m_0/m_0 = 3/(32\pi S)|H_{\text{eff}}^K/H_{\text{ex}}|
\]

in the case of easy-plane anisotropy. For the special case where \( H_{\text{eff}}^K = -H_d \) (zero magnetocrystalline anisotropy), this is in agreement with Maleev's result. For usual values of \( H_{\text{eff}}^K \) and \( H_{\text{ex}} \), the ground-state spin reduction is smaller than \( 10^{-3} \) and can therefore be neglected.

VI. TEMPERATURE DEPENDENCE OF THE MAGNETIZATION

The magnetization reduction due to thermally activated spin waves is given by formula (23). The convergence or divergence of the integral is determined by the spin-wave spectrum in the neighborhood of \( q = 0 \). We will emphasize that, not only the spin-wave energy \( \hbar \omega_q \) but also the associated spin deviation \( D_q \), are to be considered.

In order to distinguish the respective importance of the (effective) anisotropy and the long-range character of dipolar interaction, we will first "switch off" this long-range character by neglecting, in Eqs. (24a)-(25b), the terms which are linear in \( q \); by doing so, we still retain the anisotropy arising from dipolar interactions (dемагнитizing field), and the corresponding approximation will be referred to as the effective anisotropy approximation. Next, we fully take into account the long-range character of dipolar interactions. Once the convergence of the integral has been established, we evaluate it approximately by replacing the first Brillouin zone by a disk of equal area, and retaining only the most significant terms.

A. Perpendicular easy axis

For the case of perpendicular easy axis, the effective anisotropy approximation reduces to the Döring theory: the spin-wave energy and associated spin deviation are given by

\[
\hbar \omega_q = 2 \mu_B(H_{\text{eff}}^K + H_{\text{ex}}q^2),
\]

\[
D_q = 1.
\]

One thus obtains for the relative magnetic moment variation

\[
\Delta m(T)/m_0 = 1/(4\pi S \mu_B H_{\text{ex}})^2 \ln \left[ 1 - \exp \left( -\frac{2 \mu_B H_{\text{eff}}^K}{\hbar T} \right) \right]^{-1},
\]

which reduces to

\[
\Delta m(T)/m_0 = \frac{1}{4\pi S \mu_B H_{\text{ex}}} \frac{k_B T}{2 \mu_B H_{\text{ex}}} \exp \left( -\frac{2 \mu_B H_{\text{eff}}^K}{\hbar T} \right),
\]

for \( k_B T \ll 2 \mu_B H_{\text{eff}}^K \), and to

\[
\Delta m(T)/m_0 = \frac{1}{4\pi S \mu_B H_{\text{ex}}} \frac{k_B T}{2 \mu_B H_{\text{eff}}^K},
\]

for \( k_B T \gg 2 \mu_B H_{\text{eff}}^K \).
for \( k_B T > 2 \mu_B H_{K}^{\text{eff}} \). As Eq. (34) shows, the Nernst theorem stating that \( dm / dT \to 0 \) as \( T \to 0 \) is satisfied.

If we now take into account the long-range character of dipolar interactions and retain only the leading terms, we obtain the same results as given by Eqs. (34) and (35). This shows that the stabilization of the magnetization in this case is due mainly to the positive value of the effective anisotropy and to the associated gap at the bottom of the spin-wave spectrum; the role of the long-range character of dipolar interactions turns out to be negligible, at least as long as the effective anisotropy field \( H_{K}^{\text{eff}} \) is large compared to \( H_{\text{ex}} \), which is the usual situation.

### B. Easy-plane anisotropy

For the case of easy-plane anisotropy, in the effective anisotropy approximation, the spin-wave spectrum is linear in the neighborhood of \( q = 0 \):

\[
\tilde{\omega}_q = 2\mu_B |H_{K}^{\text{eff}}|^{1/2} H_d^{1/2} q ,
\]

whereas the spin deviation \( D_q \) diverges like \( q^{-1} \):

\[
D_q \approx \frac{1}{2} \left( |H_{K}^{\text{eff}}| / H_{\text{ex}} \right)^{1/2} q^{-1} .
\]

As a consequence, the integral giving the magnetic moment reduction diverges logarithmically at its lower bound, showing that long-range ferromagnetic order does not exist at finite temperature, within the effective anisotropy approximation.

We now consider the effect of the long-range character of dipolar interactions. As a matter of fact, the bottom of the spin-wave spectrum \( q \approx 0 \) is strongly modified by the latter; the spin-wave energy now has a square-root dispersion relation:

\[
\tilde{\omega}_q = 2\mu_B |H_{K}^{\text{eff}}|^{1/2} (H_d / \alpha)^{1/2} \times 2q^{1/2} \left[ 1 - \cos(2\phi_q) \right]^{1/2} ,
\]

whereas the spin deviation \( D_q \) only diverges like \( q^{-1/2} \):

\[
D_q \approx \frac{1}{8} \frac{H_d^{3/2}}{H_{\text{ex}} |H_{K}^{\text{eff}}|^{1/2}} q^{-1/2} \left[ 1 - \cos(2\phi_q) \right]^{-1/2} .
\]

One can then easily verify that the logarithmic divergence in \( \Delta m (T) \) is removed. The long-range character of dipolar interactions therefore appears to play a key role with respect to the stabilization of a long-range magnetic order at finite temperature.

For the evaluation of the temperature dependence of the magnetic moment, we will neglect the dependence of \( A_q \) and \( B_q \) with respect to \( \phi_q \) in Eqs. (25a) and (25b); one can easily check that this approximation does not alter the convergence of the integral. In the limit \( T \to 0 \), we obtain

\[
\Delta m (T) \approx \frac{\alpha \xi(3)}{\pi S} \left( \frac{k_B T}{\mu_B |H_{K}^{\text{eff}}|} \right)^3
\]

so that Nernst's theorem is satisfied \( [\xi(3) = 1.20 \text{ is Riemann's } \xi \text{ function}] \). For \( k_B T > 2 \mu_B |H_{K}^{\text{eff}}| \), we find the following result:

\[
\frac{\Delta m (T)}{m_0} = \frac{1}{4\pi} \frac{k_B T}{2 \mu_B |H_{K}^{\text{eff}}|} \left( \frac{k_B T}{2 \mu_B |H_{K}^{\text{eff}}|} \right) \ln \left( \frac{k_B T}{2 \mu_B |H_{K}^{\text{eff}}|} \right).
\]

It is quite remarkable that, although there is actually no gap in the spin-wave spectrum, the temperature dependence of the magnetic moment has the same form as for the case of perpendicular easy axis [Eq. (35)], but the gap energy \( E_{\text{gap}} = 2 \mu_B |H_{K}^{\text{eff}}| \) is to be replaced by a pseudogap of magnitude

\[
E_{\text{pseudogap}} = 2 \mu_B H_{\text{ex}} |H_{K}^{\text{eff}}|^{1/2} / (8\alpha H_{\text{ex}}^{1/2}) .
\]

For the case of zero magnetocrystalline anisotropy \( (H_{K}^{\text{eff}} = H_d) \), our problem reduces to the one discussed by Maleev (only dipolar and exchange interactions). For \( k_B T > 2 \mu_B |H_{K}^{\text{eff}}| \), our result (41) is in agreement with Maleev's result; on the other hand, in the limit \( T \to 0 \), he finds \( m (T) \sim T^{3/2} \), whereas we obtain \( m (T) \sim T^2 \). The latter discrepancy indicates that our approximation neglecting the angular-dependent terms in Eqs. (38) and (39) is rather poor at very low temperature; it is a very good one, however, in the temperature range of experimental interest \( k_B T > 2 \mu_B |H_{K}^{\text{eff}}| \).

### VII. DISCUSSION

It is often argued in the literature that the magnetization of two-dimensional ferromagnets should decrease linearly with the temperature;\(^{15-19}\) some authors even claimed that the observation of such a linear behavior constitutes a proof of the truly two-dimensional nature of the system.\(^{16}\) On the other hand, some other authors conjectured that two-dimensional ferromagnets with a perpendicular easy axis should present a linear temperature dependence of the magnetization, whereas two-dimensional ferromagnets with easy-plane anisotropy should not.\(^{20}\)

Actually, apart from the objection that such a linear dependence violates the Nernst theorem, the above calculations show that, for both the cases of perpendicular-easy-axis and of easy-plane anisotropy, the temperature dependence of the magnetization is like \( T \ln (T) \), which is definitely not linear (this temperature dependence, however, does not hold in the very neighborhood of \( T = 0 \), and at elevated temperature where the spin-wave approximation breaks down). It seems more likely that, as discussed in Refs. 21 and 22, the observed linear dependence of the magnetization is related to some superparamagnetic relaxation due to an island structure.

In order to give a more quantitative idea of the predictions of the above theory in a case of experimental interest, we have performed calculations of the temperature dependence of the magnetization in a cobalt fcc (001) monolayer for different values of the effective anisotropy field \( H_{K}^{\text{eff}} \). We have assumed the bulk value for the ground-state magnetization and the exchange energy, i.e., \( H_d = 14 \text{ kOe} \) and \( H_{\text{ex}} = 1970 \text{ kOe} \), respectively. Figure 1 shows the results obtained for \( H_{K}^{\text{eff}} = 2, 14, \) and 100 kOe, respectively (perpendicular easy axis), and \( H_{K}^{\text{eff}} = -2, -14, \) and -100 kOe (easy-plane anisotropy). These results have been obtained by numerical integration of Eqs.
One clearly observes that, for a given absolute value of the effective anisotropy field, the magnetization is much smaller than in the ground state; the Curie temperature is probably in the range of 200 K, and certainly well below room temperature. The magnetic properties of ultrathin fcc (001) Co films epitaxially grown onto (001) Cu single crystals have been extensively studied in Jülich and in Berlin, and very controversial results have been obtained.

(i) The Jülich group found that, in a fcc (001) Co monolayer, the spontaneous magnetization remains equal to its ground-state value (within experimental accuracy, i.e., 10%) up to temperatures as large as 400 K, so that the Curie temperature is probably above 500 K.

(ii) On the other hand, the Berlin group studied the thickness dependence of the Curie temperature in fcc (001) Co ultrathin films and found that, for one atomic layer of Co, the Curie temperature is lower than 100 K.

(iii) Both authors, however, report that the spontaneous magnetization lies in the film plane; with our notations, this corresponds to $H_K^{\text{eff}} < 0$.

In order to interpret their surprising result, the Jülich authors suggested that the dipolar interactions might be responsible for the stabilization of the long-range magnetic order up to a temperature of about 500 K. This interpretation is in strong contradiction with our theoretical results; rather, it seems more likely that, as suggested by the Berlin group, the monolayer investigated in Jülich was actually thicker, due to an erroneous thickness calibration.

Clearly, more experimental and theoretical investigations are required in order to understand the temperature dependence of the magnetization in two-dimensional ferromagnets.

ACKNOWLEDGMENTS

I am grateful to Dr. G. Bayreuther and Dr. J. Seiden for helpful criticism and suggestions, and to Professor U. Krey for very instructive discussions. I am pleased to acknowledge the financial support of the Alexander von Humboldt Foundation.